

Accelerating universe in $f(\mathcal{R})$ brane gravity

K. Atazadeh*, M. Farhoudi[†] and H. R. Sepangi[‡]

Department of Physics, Shahid Beheshti University, Evin, Tehran 19839, Iran

October 10, 2007

Abstract

We study a 5-dimensional $f(\mathcal{R})$ brane gravity within the framework of scalar-tensor type theories. We show that such a model predicts, for a certain choice of $f(\mathcal{R})$ and a spatially flat universe, an exponential potential, leading to an accelerated expanding universe driven solely by the curvature of the bulk space. This result is consistent with the observational data in the cosmological scale.

1 Introduction

The idea that our world might be a brane embedded in a higher dimensional space-time (the bulk) [1] has been in the mainstream of cosmological investigations in the past few years [2, 3]. This approach differs from the usual Kaluza-Klein idea in that the size of the extra dimensions can be large. The concept of large extra dimensions is discussed phenomenologically in [4]. An important ingredient of the brane world scenario is that the matter is confined to the brane and the only communication between the brane and bulk is through gravitational interaction or some other dilatonic matter. In general, the matter on the brane leads to a cosmological evolution which is different from the usual evolution governed by the Friedmann equation, that is, in brane cosmology the Hubble parameter on the brane is proportional to the square of energy density [2, 3]. This proportionality is a result of the application of the Israel matching condition which is basically a relation between the extrinsic curvature and the energy-momentum tensor representing matter fields on the brane.

Although in brane theories matter fields live on the brane, the possibility of the presence of matter in the form of a scalar field in the bulk has also been investigated in several works. One of the first motivations to introduce a bulk scalar field was to stabilize [5] the distance between the two branes in the context of the first model introduced by Randall and Sundrum [1]. A second motivation was the possibility of the resolution of the famous cosmological constant problem [6]. Several works have studied, in particular, the impact of the presence of a scalar field in the bulk on the cosmological evolution on the brane, without trying to solve the full system of equations in the bulk [7, 8]. In [9], the authors have addressed some of the solutions for these equations and studied the corresponding brane evolution. The purpose of the present study is to employ modified gravity [10] in the Einstein frame to explain the origin of such a self interacting scalar potential.

*email: k-atazadeh@sbu.ac.ir

[†]email: m-farhoudi@sbu.ac.ir

[‡]email: hr-sepangi@sbu.ac.ir

An interesting observation made a few years ago was that the expansion of our universe is currently undergoing a period of acceleration which is directly measured from the light curves of several hundred type Ia supernovae [11] and independently from observations of the cosmic microwave background (CMB) by the WMAP satellite [12] and other CMB experiments [13]. However, the mechanism responsible for this acceleration is not well understood and many authors introduce a mysterious cosmic fluid, the so called dark energy, to explain this effect [14]. Recently, it has been shown that such an accelerated expansion could be the result of a modification to the Einstein-Hilbert action [15] in the framework of DGP brane cosmology. In the present work we study the general form of the Einstein-Hilbert action for any function of the Ricci scalar, $f(\mathcal{R})$, in 5 dimensions. This is done in the framework of a scalar-tensor type theory [16] where a scalar field is minimally coupled to gravity with a self-interacting potential. In this formulation we obtain explicit solutions using conformal transformations, a technique employed in the case of an empty bulk with a cosmological constant [17] or a bulk with a scalar field, similar to the present work, but with an exponential potential. We present explicit solutions for a particular choice of $f(\mathcal{R})$ which predict a similar exponential potential.

The organization of the manuscript is as follows: in section 2 we briefly review the scalar-tensor formulation in 5-dimensions and write the full system of equations. In section 3 we consider the cosmological equations for $f(\mathcal{R})$ gravity which, in the Einstein frame, correspond to a self interacting scalar field with a certain potential. Finally, we study the cosmological evolution on the brane for \mathcal{R}^m gravity which predicts a power law acceleration in section 4. Conclusions are drawn in the last section.

2 Scalar-Tensor formulation of $f(\mathcal{R})$ gravity

Let us start from a general 5-dimensional action in the matter frame

$$\mathcal{S}[g_{AB}] = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} f(\mathcal{R}) + \mathcal{S}_m[q_{\mu\nu}, \psi_m]. \quad (1)$$

Here, $\kappa_5^2 = 8\pi G_5$, \mathcal{R} is the 5-dimensional scalar curvature and $f(\mathcal{R})$ is some arbitrary function of the scalar curvature with \mathcal{S}_m being the matter action defined by the induced metric $q_{\mu\nu}$ and the matter field ψ_m on the brane. Under the conformal transformation [18]

$$\tilde{g}_{AB} = e^{\frac{\kappa_5}{\sqrt{3}}\Phi} g_{AB}, \quad (2)$$

and the choice

$$\Phi = \frac{2}{\sqrt{3}\kappa_5} \ln f'(\mathcal{R}), \quad (3)$$

where the prime denotes derivative with respect to \mathcal{R} , action (1) can be written in the Einstein frame as [16]

$$\tilde{\mathcal{S}}[\tilde{g}_{AB}, \Phi] = \int d^5x \sqrt{-\tilde{g}} \left[\frac{\tilde{\mathcal{R}}}{2\kappa_5^2} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - V(\Phi) \right] + \tilde{\mathcal{S}}_m[\tilde{q}_{\mu\nu}, \psi_m], \quad (4)$$

where \tilde{g}_{AB} and $\tilde{q}_{\mu\nu} = e^{\frac{1}{\sqrt{3}}\kappa_5\Phi} q_{\mu\nu}$ are the 5D bulk metric and the induced metric on the brane in the Einstein frame respectively and $\tilde{\mathcal{R}}$ is the 5D Ricci scalar associated with \tilde{g}_{AB} . One can show that the effective potential in 5D is given by

$$V(\Phi) = \frac{\mathcal{R}f'(\mathcal{R}) - f(\mathcal{R})}{2\kappa_5^2 f'(\mathcal{R})^{5/3}}. \quad (5)$$

This is the standard form of the scalar-tensor type theories mentioned above. The 5D equations of motion corresponding to action (4) are

$$\tilde{G}_{AB} = \kappa_5^2 \left[\tilde{T}_{AB}(\psi_m, \tilde{g}) + \tilde{\mathcal{T}}_{AB}(\Phi, \tilde{g}) \right], \quad (6)$$

where \tilde{T}_{AB} is the energy-momentum tensor in the Einstein frame and \tilde{T}_{AB} is given by

$$\tilde{T}_{AB} = \partial_A \Phi \partial_B \Phi - \tilde{g}_{AB} \left[\frac{1}{2} (\tilde{\nabla}_C \Phi) (\tilde{\nabla}^C \Phi) + V(\Phi) \right], \quad (7)$$

with the equation of motion for the scalar field as

$$\tilde{\nabla}_A \tilde{\nabla}^A \Phi - \frac{dV(\Phi)}{d\Phi} = \frac{\kappa_5}{2\sqrt{3}} e^{-\frac{2\kappa_5}{\sqrt{3}}\Phi} T \delta(y), \quad (8)$$

where T is the trace of energy momentum tensor in the Jordan frame and y represents the extra dimension. For cosmological considerations, let us take a general form for the bulk metric in the matter frame, also known as the Jordan frame, usually assumed as

$$ds^2 = g_{AB} dx^A dx^B = -n^2(y, t) dt^2 + a^2(y, t) \gamma_{ij} dx^i dx^j + b^2(y, t) dy^2, \quad (9)$$

where γ_{ij} is the maximally symmetric 3-dimensional metric with $k = -1, 0, 1$ being the usual parameters denoting the spatial curvature. Therefore, in the Einstein frame the metric is \tilde{g}_{AB} and the functions $b(y, t)$, $a(y, t)$ and dt can be written as

$$\tilde{b}(y, \tilde{t}) = e^{\frac{\kappa_5}{2\sqrt{3}}\Phi} b(y, t), \quad (10)$$

$$\tilde{a}(y, \tilde{t}) = e^{\frac{\kappa_5}{2\sqrt{3}}\Phi} a(y, t) \quad (11)$$

and

$$d\tilde{t} = e^{\frac{\kappa_5}{2\sqrt{3}}\Phi} dt. \quad (12)$$

Let us also take the matter on the brane as a perfect fluid, given by

$$\tilde{T}^A{}_B = \frac{1}{\tilde{b}(y, \tilde{t})} \text{diag}[\tilde{\rho}(\tilde{t}), \tilde{p}(\tilde{t}), \tilde{p}(\tilde{t}), \tilde{p}(\tilde{t}), 0] \delta(y), \quad (13)$$

where

$$\tilde{\rho} = e^{-\frac{2\kappa_5}{\sqrt{3}}\Phi} \rho, \quad (14)$$

$$\tilde{p} = e^{-\frac{2\kappa_5}{\sqrt{3}}\Phi} p. \quad (15)$$

Here, $\tilde{\rho}$ and \tilde{p} respectively are the energy density and pressure in the Einstein frame. In the Gauss normal coordinates, $\tilde{g}_{55} = \tilde{b}^2(y, \tilde{t}) = 1$, the 5-dimensional bulk equations (6) can be written as

$$3 \left\{ \left(\frac{\dot{\tilde{a}}}{\tilde{a}} \right)^2 - n^2 \left[\frac{\tilde{a}''}{\tilde{a}} + \left(\frac{\tilde{a}'}{\tilde{a}} \right)^2 \right] + k \frac{n^2}{\tilde{a}^2} \right\} = \kappa_5^2 \left[n^2 V(\Phi) + \frac{1}{2} \dot{\Phi}^2 + \frac{n^2}{2} \Phi'^2 + n^2 \tilde{\rho} \delta(y) \right], \quad (16)$$

$$3 \left(\frac{n'}{n} \frac{\dot{\tilde{a}}}{\tilde{a}} - \frac{\dot{\tilde{a}}'}{\tilde{a}} \right) = \kappa_5^2 \dot{\Phi} \Phi', \quad (17)$$

$$3 \left\{ \frac{\tilde{a}'}{\tilde{a}} \left(\frac{\tilde{a}'}{\tilde{a}} + \frac{n'}{n} \right) - \frac{1}{n^2} \left[\frac{\dot{\tilde{a}}}{\tilde{a}} \left(\frac{\dot{\tilde{a}}}{\tilde{a}} - \frac{\dot{n}}{n} \right) + \frac{\ddot{\tilde{a}}}{\tilde{a}} \right] + \frac{k}{\tilde{a}^2} \right\} = \kappa_5^2 \left[V(\Phi) + \frac{1}{2n^2} \dot{\Phi}^2 + \frac{1}{2} \Phi'^2 \right] \quad (18)$$

and

$$\begin{aligned} \tilde{a}^2 \left[\frac{\tilde{a}'}{\tilde{a}} \left(\frac{\tilde{a}'}{\tilde{a}} + 2 \frac{n'}{n} \right) + 2 \frac{\tilde{a}''}{\tilde{a}} + \frac{n''}{n} \right] + \frac{\tilde{a}^2}{n^2} \left[\frac{\dot{\tilde{a}}}{\tilde{a}} \left(-\frac{\dot{\tilde{a}}}{\tilde{a}} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\ddot{\tilde{a}}}{\tilde{a}} \right] - k \\ = -\kappa_5^2 \tilde{a}^2 \left[\frac{1}{2} V(\Phi) - \frac{1}{2n^2} \dot{\Phi}^2 + \Phi'^2 - \tilde{p} \delta(y) \right]. \end{aligned} \quad (19)$$

The scalar field in the bulk, equation (8), also reads

$$\ddot{\Phi} + \left(3\frac{\dot{\tilde{a}}}{\tilde{a}} - \frac{\dot{n}}{n}\right)\dot{\Phi} - n^2 \left[\Phi'' + \left(\frac{n'}{n} + 3\frac{\tilde{a}'}{\tilde{a}}\right)\Phi'\right] + n^2 \frac{dV(\Phi)}{d\Phi} = n^2 \frac{-\kappa_5}{2\sqrt{3}} e^{\frac{-2\kappa_5}{\sqrt{3}}\Phi} T \delta(y), \quad (20)$$

where the prime and dot represent derivative with respect to y and \tilde{t} respectively.

Assuming \mathbf{Z}_2 symmetry and denoting $\tilde{a}_0(\tilde{t}) \equiv \tilde{a}(0^+, \tilde{t})$, $\tilde{a}'_0(\tilde{t}) \equiv \tilde{a}'(0^+, \tilde{t})$, $n_0(\tilde{t}) \equiv n(0^+, \tilde{t})$, $n'_0(\tilde{t}) \equiv n'(0^+, \tilde{t})$ and $\Phi'_0(\tilde{t}) \equiv \Phi'(0^+, \tilde{t})$, we may proceed to extract from the delta functions on both sides of the equations (16) and (19), the matching conditions

$$\frac{\tilde{a}'_0}{\tilde{a}_0} = -\frac{\kappa_5^2}{6} \tilde{\rho}(\tilde{t}) \quad (21)$$

and

$$\frac{n'_0}{n_0} = \frac{\kappa_5^2}{6} [3\tilde{p}(\tilde{t}) + 2\tilde{\rho}(\tilde{t})]. \quad (22)$$

One notes that equation (21) is consistent with the assumption that the effect of extra dimension diminishes as one moves away from the brane. Let us now turn to matching condition for the scalar field. Using (20) we obtain

$$2\Phi'_0 = \frac{\kappa_5}{2\sqrt{3}} e^{-\frac{2\kappa_5}{\sqrt{3}}\Phi_0} T^{(\text{brane})}, \quad (23)$$

where $T^{(\text{brane})} = -\rho + 3p$ is the trace for the energy-momentum tensor in the matter frame. Application of the matching condition for the scalar field leads to

$$\Phi'_0 = \frac{\kappa_5}{2\sqrt{3}} \gamma \tilde{\rho}(\tilde{t}), \quad (24)$$

which involves all cases where the equation of state is of the form $\tilde{p} = w\tilde{\rho}$ with w as a constant and the expression for γ given by

$$\gamma = \frac{1}{2}(3w - 1). \quad (25)$$

If the Lagrangian density for the perfect fluid is proportional to the pressure as chosen in [8]

$$\mathcal{L}(\phi) = -2\mathcal{F}(\phi)p(s, \varepsilon), \quad (26)$$

where $\mathcal{F}(\phi)$ is an arbitrary function, s and ε are entropy and enthalpy respectively, then in this model we will have $\gamma = -4w\chi$. Note that in our model $\chi = \frac{1}{2}$. Thus, if the matter content of the brane behaves like a cosmological constant, $w = -1$, this model will be compatible with that presented in [8].

3 Cosmological equations on the brane

In this section we consider the cosmological behavior on the brane using the global equations obtained in the previous section. For the brane, assumed to stay at $y = 0$, the Einstein frame induced FRW metric with $k = 0$ is

$$d\tilde{s}^2 = -n_0^2(\tilde{t})d\tilde{t}^2 + \tilde{a}_0^2(\tilde{t})\delta_{ij}dx^i dx^j. \quad (27)$$

One may now proceed to obtain the cosmological equations by taking the gauge

$$n_0(\tilde{t}) = 1. \quad (28)$$

The cosmic time τ in the Einstein frame can be derived from the \tilde{t} by

$$\tau = \int^{\tilde{t}} n_0(t') dt'. \quad (29)$$

This gauge is convenient because it gives the usual cosmological time on the brane.

Let us now obtain the Friedmann equation as well as a generalized conservation equation on the brane. We will closely follow the derivation presented in [3, 9] with the additional ingredient of an energy flux from the fifth dimension, *i.e.* the component (0,5) of the bulk energy-momentum tensor is assumed to be non zero because of the presence of the scalar field. Use of the matching conditions in the (0,5) component of the field equations, (17), in the Einstein frame evaluated on the brane yields the generalized conservation equation

$$\dot{\tilde{\rho}} + 3\frac{\dot{\tilde{a}}_0}{\tilde{a}_0}(\tilde{\rho} + \tilde{p}) = 2\tilde{T}_{05}\Big|_{y=0}, \quad (30)$$

where $\tilde{T}_{05} = \dot{\Phi}\Phi'$. Now, by using the matching condition (24) for the scalar field, it reads

$$\dot{\tilde{\rho}} + 3\frac{\dot{\tilde{a}}_0}{\tilde{a}_0}(\tilde{\rho} + \tilde{p}) = \gamma\dot{\Phi}_0\tilde{\rho}, \quad (31)$$

where $\bar{\Phi} \equiv \frac{\kappa_5}{\sqrt{3}}\Phi$ and the dot represents derivative with respect to τ . This equation is the generalized conservation law for cosmological matter. For an equation of state $\tilde{p} = w\tilde{\rho}$ with w constant, the integration of equation (31) yields the following evolution for the energy density

$$\tilde{\rho} \propto \tilde{a}_0^{-3(1+w)} e^{\gamma\bar{\Phi}_0}. \quad (32)$$

If the scalar field is constant in time we will recover the familiar evolution of the standard cosmology. Let us now consider the (5,5) component of the field equations, (18). Using the (0,5) component, it can be rewritten in the form

$$\dot{F} = \frac{2}{3}\dot{\tilde{a}}_0\tilde{a}_0^3\kappa_5^2\tilde{T}^5{}_5\Big|_{y=0} - \frac{2}{3}\tilde{a}_0'\tilde{a}_0^3\kappa_5^2\tilde{T}^5{}_0\Big|_{y=0}, \quad (33)$$

with

$$F \equiv (\tilde{a}_0\tilde{a}_0')^2 - (\tilde{a}_0\dot{\tilde{a}}_0)^2. \quad (34)$$

This corresponds to a slight generalization of the expression given in [3]. The expression for the (5,5) component of the energy-momentum tensor of the scalar field \tilde{T}_{AB} is given by

$$\tilde{T}^5{}_5 = \frac{1}{2}(\Phi'^2 + \dot{\Phi}^2) - V(\Phi). \quad (35)$$

Using equation (33) and the matching conditions, one obtains, after integrating the time, the following generalized Friedmann equation in the Einstein frame

$$\tilde{H}_0^2 = \frac{\kappa_5^4}{36}\tilde{\rho}^2 - \frac{2\kappa_5^2}{3\tilde{a}_0^4}\int d\tau\dot{\tilde{a}}_0\tilde{a}_0^3\tilde{T}^5{}_5\Big|_{y=0} - \frac{\kappa_5^4\gamma}{18\tilde{a}_0^4}\int d\tau\dot{\Phi}_0\tilde{a}_0^4\tilde{\rho}^2, \quad (36)$$

where the Hubble parameter is defined by

$$\tilde{H}_0 \equiv \frac{\dot{\tilde{a}}_0}{\tilde{a}_0}, \quad (37)$$

and the constant of integration is taken to be zero. The quadratic appearance of the energy density in this equation is a generic feature of the brane cosmology [2]. It also has an integral term related to the pressure along the fifth dimension and an integral term related to the energy flux coming from the bulk scalar field.

Now, using the matching condition (24) we obtain

$$\frac{2}{3}\kappa_5^2\tilde{T}^5{}_5\Big|_{y=0} = \frac{\kappa_5^4}{36}\gamma^2\tilde{\rho}^2 + \dot{\Phi}_0^2 - \frac{2}{3}\kappa_5^2V(\Phi)\Big|_{y=0}. \quad (38)$$

Finally, after evaluating (20) at $y = 0$ together with the use of the matching conditions (21), (22) and (24) the scalar field equation on the brane is given by

$$\ddot{\Phi}_0 + 3 \left(\frac{\dot{\tilde{a}}_0}{\tilde{a}_0} \right) \dot{\Phi}_0 - \hat{\Phi}_0'' + \frac{dV(\Phi)}{d\Phi} \Big|_{y=0} = \frac{\kappa_5^3}{6\sqrt{3}} \gamma^2 \tilde{\rho}^2(\tau), \quad (39)$$

where $\hat{\Phi}_0''$ stands for the non-distributional part of the scalar field derivative. Thus equations (36) and (39) are the equations of motion for the evolution of the cosmic on the brane in the Einstein frame. In the next section we will examine these equations for a particular choice of $f(R)$ gravity.

4 Cosmological evolution in \mathcal{R}^m gravity

We start from the Lagrangian

$$f(\mathcal{R}) = f_0 \mathcal{R}^m, \quad (40)$$

for which potential (5) is given by

$$V(\Phi) = V_0 e^{\alpha \bar{\Phi}}, \quad (41)$$

where

$$V_0 = \frac{1}{2\kappa_5^2} f_0 (m-1) (m f_0)^{\frac{m}{1-m}}, \quad (42)$$

with $\alpha \equiv \frac{-2m+5}{2(m-1)}$ and f_0 is a constant.

As it can be seen, the exponent in the above potential is singular for $m = 1$ and therefore warrants further discussion. For this value of m , the scalar field Φ from equation (3) becomes constant and we have $\tilde{g}_{AB} = \text{const.} \times g_{AB}$, indicating that the Jordan frame is equivalent to the Einstein frame. Also, the effective potential for $m = 1$ in the Einstein frame is zero, similar to what one obtains in an empty $4D$ universe for which the dynamics is governed by the same Lagrangian [16, 18]. This seems to be a general feature of modified theories of gravity when the Lagrangian is of the form (40). In what follows, we determine the range of validity for m which would allow the universe to achieve an accelerated expansion.

Now, we assume that both the total energy density ρ and pressure p on the brane consist of two parts

$$\rho = \lambda + \varrho \quad \text{and} \quad p = -\lambda + \mathbf{p}, \quad (43)$$

where λ , ϱ and \mathbf{p} are the tension, the usual cosmological energy density and pressure in the matter frame, respectively. In what follows we concentrate on the case $\varrho = \mathbf{p} = 0$, *i.e.* the vacuum solution. Equation (25) then implies that $\gamma = -2$. One notes that by retaining a non-zero effective tension on the brane we are actually taking the brane effects into account. For simplicity and following [19], we take the tension, λ , in the matter frame as

$$\lambda = \bar{\lambda}_c e^{\left(\frac{\alpha}{2}+2\right)\bar{\Phi}}, \quad (44)$$

where $\bar{\lambda}_c \equiv \frac{\lambda_c}{\kappa_5^2}$ and λ_c is a constant. Therefore, $\tilde{\lambda} = e^{-2\bar{\Phi}} \lambda = \bar{\lambda}_c e^{\frac{\alpha}{2}\bar{\Phi}}$ is the brane tension in the Einstein frame. Thus, the equations of motion on the brane, (36) and (39), become

$$\tilde{H}_0^2 = \frac{\kappa_5^4}{36} \tilde{\lambda}^2 - \frac{2\kappa_5^2}{3\tilde{a}_0^4} \int d\tau \dot{\tilde{a}}_0 \tilde{a}_0^3 \tilde{T}^5 \Big|_{y=0} - \frac{\kappa_5^4}{9\tilde{a}_0^4} \int d\tau \tilde{\lambda} \tilde{a}_0^4 \tilde{T}^5 \Big|_{y=0} \quad (45)$$

and

$$\ddot{\Phi}_0 + 3 \left(\frac{\dot{\tilde{a}}_0}{\tilde{a}_0} \right) \dot{\Phi}_0 - \hat{\Phi}_0'' + \frac{dV(\Phi)}{d\Phi} \Big|_{y=0} = \frac{2\kappa_5^3}{3\sqrt{3}} \tilde{\lambda}^2. \quad (46)$$

These equations are the basic equations of motion on the brane without matter in the Einstein frame. We now look for a power law solution for the scale factor. Substituting the ansätze

$$\tilde{a}_0(\tau) \propto \tau^\beta \quad \text{and} \quad \Phi_0(\tau) = \sigma \ln \tau \quad (47)$$

into equations (45) and (46), we find

$$\sigma = -2 \frac{\sqrt{3}}{\kappa_5 \alpha}, \quad (48)$$

where $\tau \neq 0$ and $\alpha \neq 0$, *i.e.* $m \neq 5/2$. Now, using the above value for σ into equations (45) and (46) we have

$$(4\beta - 2)\beta^2 - \left(\frac{4\beta}{9} - \frac{2}{9\alpha} - \frac{1}{18} \right) \lambda_c^2 + \frac{4(2\beta - 3\beta^2)}{\alpha^2} = 0. \quad (49)$$

This algebraic equation has one explicit real solution for β in terms of α and λ_c . To obtain the functional dependence of τ , we note that it is the cosmic time in the Einstein frame which is related to coordinate t in the matter frame by $e^{-\frac{1}{2}\Phi_0} d\tau = dt$. As a result

$$\tau = \left(\frac{\alpha + 1}{\alpha} \right)^{\frac{\alpha}{\alpha+1}} t^{\frac{\alpha}{\alpha+1}}, \quad (50)$$

up to a constant of integration, noting that α cannot take the value -1 by definition. The scale factor in the physical (Jordan or matter) frame is thus given by

$$a_0(t) = e^{-\frac{1}{2}\Phi_0} \tilde{a}_0(\tau) \propto \left(\frac{\alpha + 1}{\alpha} \right)^{\frac{\alpha\beta+1}{\alpha+1}} t^{\frac{\alpha\beta+1}{\alpha+1}}. \quad (51)$$

Equation (51) shows that there is a possibility of having an accelerated expanding universe for some choices of m and λ_c .

The deceleration parameter on the brane as a function of m and λ_c is therefore given by

$$q(m, \lambda_c) = -\frac{a_0 \ddot{a}_0}{\dot{a}_0^2} = -\frac{\alpha\beta - \alpha}{\alpha\beta + 1}. \quad (52)$$

The condition for acceleration, $q(m, \lambda_c) < 0$, in equation (52) leads to $\beta > 1$ from which, using definition $w_{\text{eff}} = -1 - \frac{2\dot{H}_0}{3H_0^2}$ for the effective quintessence, we find $w_{\text{eff}} < -1/3$. Figure 1 shows the behavior of the deceleration parameter, q , as a function of m and λ_c . As it can be seen, for $m \rightarrow -\infty$ and $\lambda_c \rightarrow \pm\infty$ we have $q \rightarrow -1$, that is the universe finally approaches the eternal de Sitter phase. The range of validity of m shown in figure 1 is consistent with the observational SNeIa data in 4-dimensional $f(\mathcal{R})$ -models [20]. It is therefore plausible that modified gravity within the context of brane theories presents an alternative to dark energy with the possibility of having an accelerated expanding universe.

A point worth emphasizing again is that, the universe in our model, taken to be devoid of ordinary matter, would undergo an accelerated expansion for all values of λ_c if the value for m is within the range shown in figure 1 which excludes the value $m = 1$ as well. For this value of m , the two frames, namely the Jordan and Einstein frames coincide and $V(\Phi) = 0$. As was mentioned above, the same behavior is also manifest in 4-dimensional $f(\mathcal{R})$ -models where the universe is taken to be empty [16, 18]. This points to a typical behavior in $f(\mathcal{R}) \sim \mathcal{R}^m$ theories, both in four and five dimensions, where for $m = 1$ the resulting universe in the present context is a static one.

5 Conclusions

In this manuscript we have obtained explicit solutions in a brane world scenario where an arbitrary function of the Ricci scalar is taken as the bulk Lagrangian. Using a conformal transformation, the action is converted to that of a scalar-tensor type theory with a scalar field. We have shown that with a suitable choice for the function $f(\mathcal{R})$ and brane tension λ , an accelerated expanding universe emerges. The source of this acceleration is not related to an exotic matter but to a scalar field whose origin can be traced back to geometry of the brane and, specifically, to the curvature scalar \mathcal{R} and depends on two free parameters, namely α and λ_c . Hence, an accelerating universe driven by curvature would certainly seem to be a possibility.

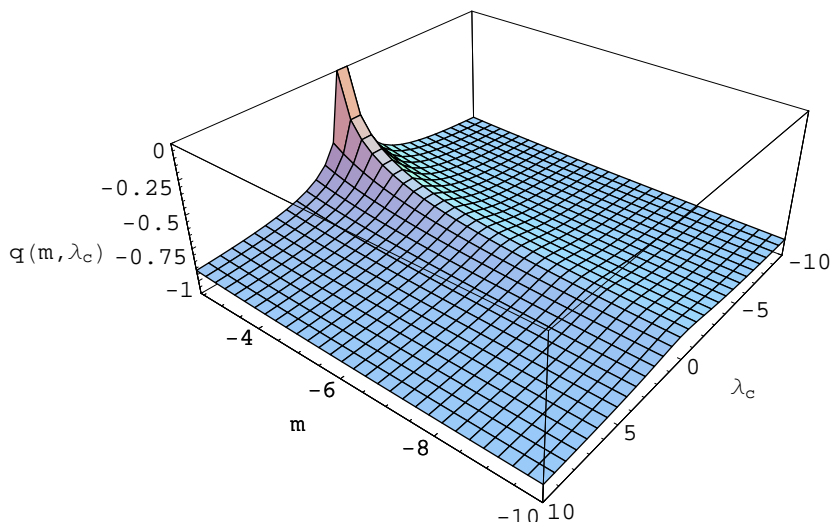


Figure 1: Behavior of $q(m, \lambda_c)$ as a function of m and λ_c . An accelerating universe occurs for $m \leq -2.42$ and for all values of λ_c .

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